Lebesgue Measure of a Set

- **Measure of an open interval** :→ The measure of any open interval G, is defined as its length and is denoted by the symbol $m(G)$.
- **Measure of Closed Interval** :→ Let $[a,b]$ be the smallest closed interval containing a closed set F. Then we define $m(F) = b - a - m(F')$.

F' being the compliment of F, w.r.t. the interval.

To prove that the measure of a closed interval is its length.

Let $A = [a,b]$. Then A is a closed interval and hence a closed set $[a,b]$ is the smallest closed interval containing A.

Complement of A relative to $[a,b] = A' \neq \phi$. For $A = [a,b]$.

$\therefore$ By definition, $m(A) = b - a - m(A') = b - a - m(\phi) = b - a$

$$= 0.$$

$$= b - a, \quad \text{i.e., } m([a,b]) = b - a.$$

- **Measure of a rectangle** :→ The area of an open rectangle $R(a < x < b,\ c < y < d)$, i.e., $(b-a)(d-c)$ is defined as the measure of R.

Thus, $m(R) = (b-a)(d-c)$.

The area of closed rect. $R(a \le x \le b,\ c \le y \le d)$.

i.e., $(b-a)(d-c)$ is defined as the measure of R.

Thus, $m(R) = (b-a)(d-c)$